Trade Secrets of a Guy with a Network Analyzer
Madison Broadcaster's Clinic 2004 Presentation

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Additional Example of Calculations Using Complex Impedance

This sample calculation demonstrates how the VSWR, Reflection Coefficient and Return Loss for a particular system may be calculated. In the example during the presentation, it was assumed that the load impedance of the system was a pure resistance, which doesn't occur in practice very often, if at all.

Problem: Calculate the VSWR, Reflection Coefficient and Return Loss for a system where the load impedance is 75+j22 Ohms and the nominal impedance of the system is 50+j0 Ohms. Assume no effects from the transmission line.

We begin this example by calculating the reflection coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{(75 + j22) - (50 + j0)}{(75 + j22) + (50 + j0)}$$

$$\Gamma = \frac{(75 - 50) + (j22 - j0)}{(75 + 50) + (j22 + j0)}$$

$$\Gamma = \frac{25 + j22}{125 + j22}$$

Now that we have the fraction, the quotient of which is the reflection coefficient for the given system, it is necessary to convert the numerator and denominator values to polar notation for ease in solving. To do this conversion, we utilize Pythagoras' Theorem and the tangent trigonometric identity. The magnitude of the numerator and denominator of the quotient is the hypotenuse of the right triangle formed by the real and imaginary terms. The arctangent of the quantity of the opposite (imaginary term) divided by the adjacent (real term) gives us the angle.

For the numerator:

$$H^2 = O^2 + A^2$$
$$H^2 = (22)^2 + (25)^2$$
$$H^2 = 1109$$
$$H = 33.30$$

Now we know the magnitude of the numerator. Lets calculate the angle.

$$\tan\theta = \frac{22}{25} = 0.88$$
$$\theta = 41.35^\circ$$

For the denominator we follow the same procedure and obtain a magnitude of 126.92 and an angle of 9.98 degrees. This results in the reflection coefficient being the following in terms of polar notation:

$$\Gamma = \frac{33.30\angle 41.35^\circ}{126.92\angle 9.98^\circ}$$

To solve this for the reflection coefficient, we simply divide the magnitude in the numerator by the magnitude in the denominator to arrive at the magnitude. To determine the angle, the value of the angle in the denominator is subtracted from the numerator as follows:

$$\Gamma = 0.2624\angle 31.37^\circ$$

Now that we know the magnitude of the reflection coefficient, we can easily determine the VSWR and the return loss for the system. We could also convert this reflection coefficient value back to a Cartesian form if desired. To do this we would the sine and cosine identities defined during the presentation. If you are interested in doing this, I'll let you do it on your own and check your results. In Cartesian form, the value of the reflection coefficient would be 0.2240+j0.1366.

Solving for VSWR:

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$
$$S = \frac{1+0.2624}{1-0.2624}$$
$$S = \frac{1.2624}{0.7376}$$
$$S = 1.711$$

Solving for Return Loss:

$$RL = -20\log|\Gamma|$$
$$RL = -20\log(0.2624)$$
$$RL = -20*(0.5810)$$
$$RL = 11.62dB$$

Following the presentation I had a question concerning the sign of the return loss as determined through this equation as it relates to that indicated on the network analyzer. Note that on the network analyzer, the return loss is indicated by a negative number, while here we have a positive number. This is due to a subtle difference in the way the network analyzer presents the information. The actual format indicated on the HP analyzers is "log mag" or logarithmic magnitude. While this is in reality return loss, what the analyzer is saying though, is that the logarithmic magnitude of the reflected signal is xx.xx dB below the incident signal, hence the difference between the signs.

It should be noted that since the magnitude of the reflection coefficient is always equal to or less than 1.0, you would always be taking the base-10 logarithm of 1, a fractional number or 0. In the case of 1, the log is 0, therefore the return loss is 0, which goes back to our discussion of opens and shorts. If the value is 0, then the log of 0 is an undefined number, or infinity, which results in an infinite return loss, or perfect match. In the case of a fractional value, then we obtain a negative term, and since the product of two negative terms is a positive term, this is how a positive value of return loss is obtained.

If you would arrive at a negative value of return loss, which would indicate a positive term after taking the log of the reflection coefficient, then a gain would be indicated and not a loss. As a result, you may have made a calculation error if you are determining values for an antenna system, or one that is passive in nature.